

ALMA MATER STUDIORUM A.D. 1088  
UNIVERSITÀ DI BOLOGNA

# Mixed integer linear programming algorithms and mathematical models for the battery storage optimal management in demand responsive off-grid power systems with renewable integration



## Research questions and objectives

- Find a good mathematical representation for degradation issues allowing the integration of design software (i.e. Homer) with more aware battery modeling; is it possible to find a good balance between storage needs and cost reduction needs? How does this affect reliability?
- Sensitivity analyses on the battery wearing costs. When will batteries become more economical in a forecasted scenario in which diesel price will continue to increase and battery cost drop?
- Sensitivity analyses on how do the battery properties (roundtrip efficiency and kinetic battery model constants) affect the degradation process and the consequent best battery use.
- How social aspects can influence the battery usage? (Africa case with no need to meet demand)
- PV panels purchase vs battery replacement in a scenario in which PV panels cost drop.

## Main research topic

The objective is the use of the **operational research techniques** to build **mathematical models** that can be used to find the optimal hourly management of offgrid systems in order to improve the **battery lifetime**, reduce the **battery degradation** due to discharge cycles and **balancing variable output of renewable** for a sustainable energy supply. From that point of view such a model can be used to **simulate different batteries technologies** and make analyses on expected performances of batteries with different properties.

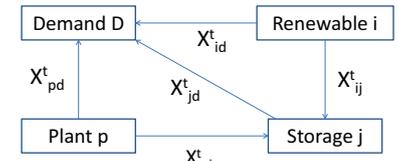
In our ideal optimization model the **battery degradation cost is a function of the energy discharged, the state of charge at the start of the cycle, the state of charge at the end of the cycle, length of the discharge event, cycles numbers and remaining cycles to failure.**

We are currently testing a **linear programming model prototype** focusing on the highest and lowest state of charge in every day as well as number of cycles and flows out the battery. The output is the optimal hourly management of the offgrid system in terms of energy flows among the following units: traditional generator, final users, PV plant, lead acid battery.

We can ask the model to tell us **how to manage the flows in the system** in such a way that:

- the lowest state of charge in every day will be reduced;
- the sum of the traditional plant costs and the battery degradation costs will be minimized;
- the number of cycles will be minimized and the greater part of cycles will end with a fully charged battery (assuming that partial cycles degrade the battery more than full cycles)

The model will be integrated with a **demand responsive formulation** that takes into account the possibility of not meeting part of the final user demand by **shifting load** of site or by programming a battery management system that limits output power at specific time of the day.



Simplified representation of the system flows

$$\text{MIN } \sum_t (X_{pd}^t + X_{pr}^t) * K_p + (X_{jd}^t * W) - \sum_g (Q_m^{g*} * W)$$

Subject to

$$\begin{aligned} X_{pd}^t + X_{id}^t + X_{jd}^t &= D_t \vee t \\ X_{pd}^t + X_{pj}^t &= 0 \quad | \quad (PP_p \leq X_{pd}^t + X_{vd}^t \leq PC_p) \vee t \\ X_{id}^t + X_{ij}^t &\leq R_t \vee t \\ Q_j^t &\geq S_j \vee t \\ Q_j^t &= Q_j^{t-1} - X_{jd}^t + X_{pj}^t + X_{ij}^t \vee t \\ X_{jd}^t &\leq A_t + Q_j^t * B \vee t \\ X_{pj}^t + X_{ij}^t &\leq H_t \vee t \\ Q_j^t &\geq Q_m^t \vee t \in g \end{aligned}$$

## PARAMETERS

$PP_p$	Minimum diesel generator production
$PC_p$	Diesel generator capacity
$R_t$	Renewable capacity
$S_j$	Battery minimum state of charge
$K_p$	Diesel generator cost per kWh
$W$	Battery wearing cost per kWh

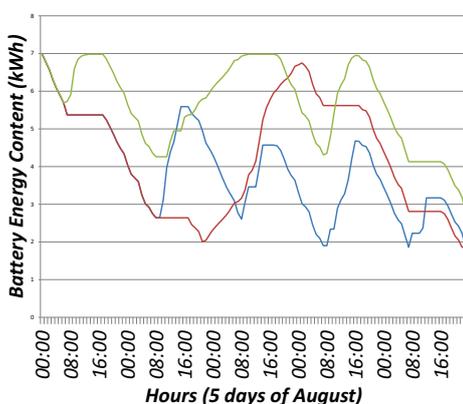
## VARIABLES

$X^t$	Energy flows (see figure above)
$Q_j^t$	Battery energy content in every time
$Q_m^t$	Lowest state of charge in a day
$A_t, B_t, H_t$	Kinetic battery model variables for the available discharge power and the allowed charge power in every time (two tanks battery model)

$$t = 1 \dots T \text{ hours}$$

$$g = 1 \dots G \text{ days}$$

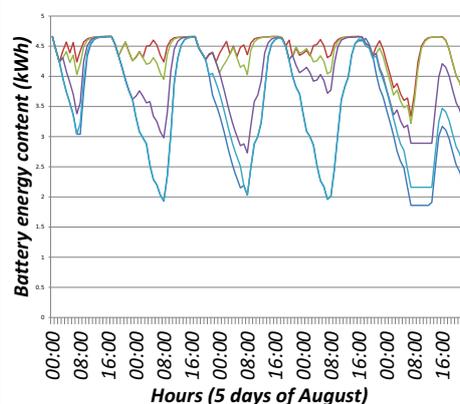
The model has been further developed adding binary variables and constraints for the number of cycles minimization and the battery energy content at the end of every single discharge cycle



—Only diesel costs minimization, no consideration of battery degradation processes

—Minimization of diesel costs, flows out the battery and number of cycles (with no consideration on the depth of discharge at the end of a cycle)

—Minimization of diesel costs, number of cycles and lowest state of charge in a day (the depth of discharge degradation is defined on a daily minimum discharge)



—Only diesel costs minimization, no consideration of battery degradation processes

—Cost of the lowest state of charge in a day equal to: [(Battery Replacement Cost) / (Throughput \* Eff)]

—Cost of the lowest state of charge in a day equal to the diesel cost

—Cost of the lowest state of charge in a day equal to 90% of the diesel cost

—Cost of the lowest state of charge in a day equal to 75% of the diesel cost

**Chiara Bordin**

(chiara.bordin2@unibo.it)

PhD student in "Automatic Control Systems and Operational Research"

**Supervised by**

**Dr. Chris Dent**

(chris.dent@durham.ac.uk)

School of Engineering and Computing Sciences  
Durham University (UK)

**Prof. Daniele Vigo**

Department of Electrical, Electronic, Information Engineering", University of Bologna (Italy)

**With a kind contribution by**

**Andrew F Crosslands**

**Harold Oghenetejiri Anuta**

(Team of the off-grid Rwanda Project,  
Durham University)