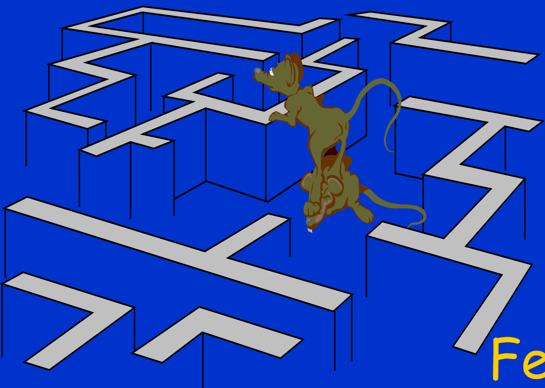
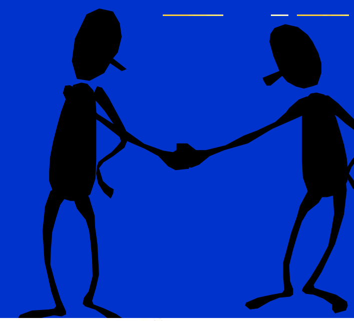
Approaches and Computational Experience in Solving the ACOPF and AC Switching





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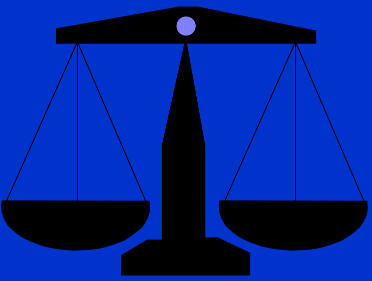
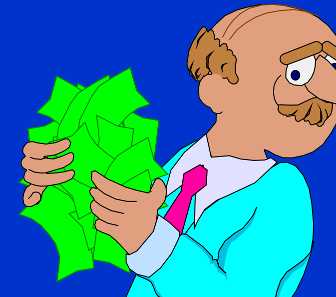
October 13, 2013

Views expressed are not necessarily those of the Commission





fictions, frictions, paradigm changes and politics

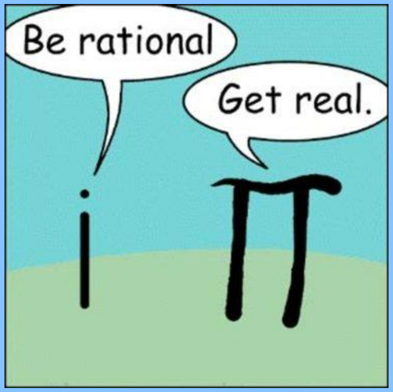
- 300 BC: Aristotle's elements
  - Air, Water, Fire, Earth, Aether
  - 'proved' voids impossible therefore no zero
  - aether fills all potential voids
- Middle Ages: Roman Church adopts Aristotle's view
  - Punished for contrary views
  - Retards the development of algebra

- 20th century: aether gradually disappears
- 21st century recycling
  - aether theory recycled as dark energy
  - Keeping zero

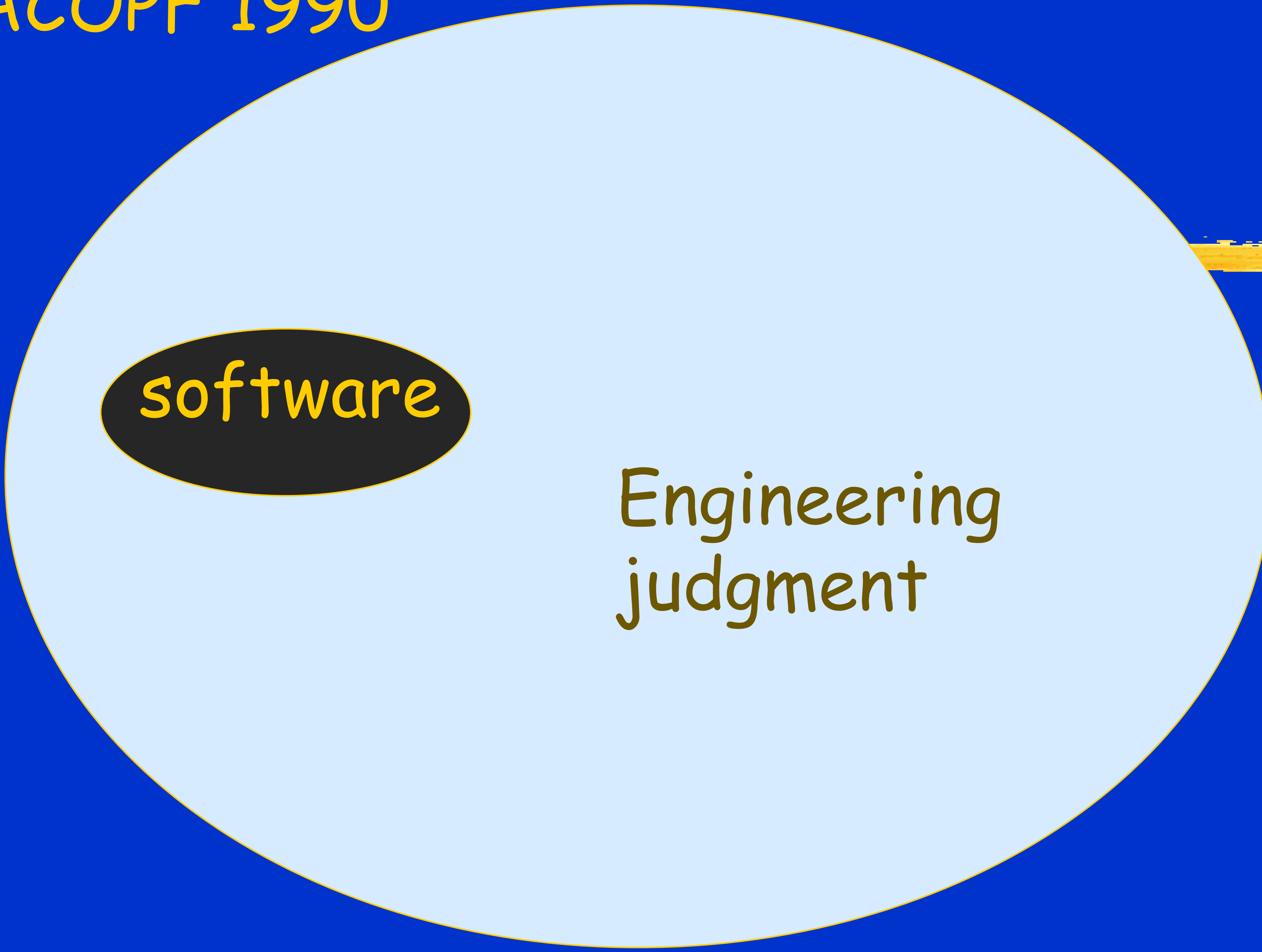
ACOPF 1960

software

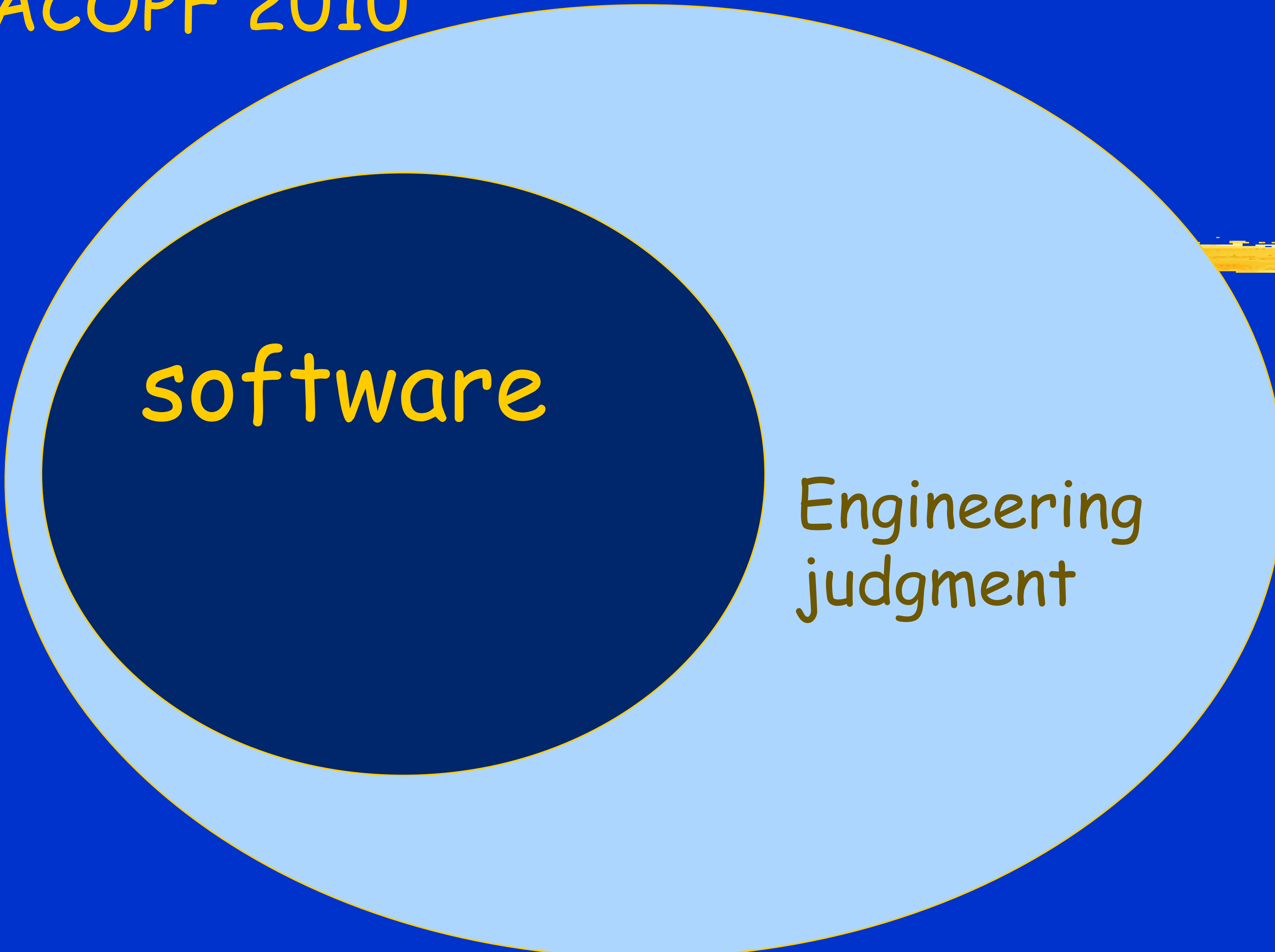
Engineering judgment



ACOPF 1990



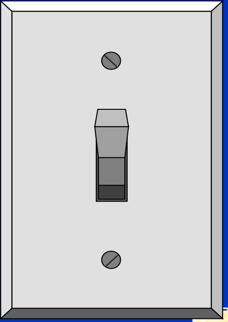
ACOPF 2010



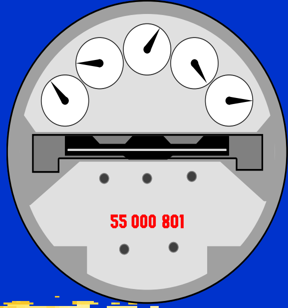
ACOPF 2020

software

Engineering
judgment



End-use markets
got to get you into my life



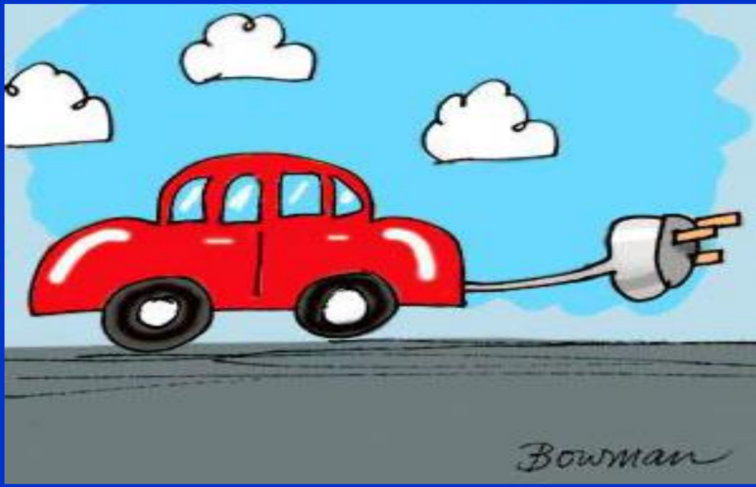
- Consumers receive very weak price signals
  - monthly meter; 'see' monthly average price
  - On a hot summer day
    - wholesale price = $1000/MWh
    - Retail price < $100/MWh
  - ☞ results in market inefficiencies and
  - ☞ poor purchase decisions for electricity and electric appliances.

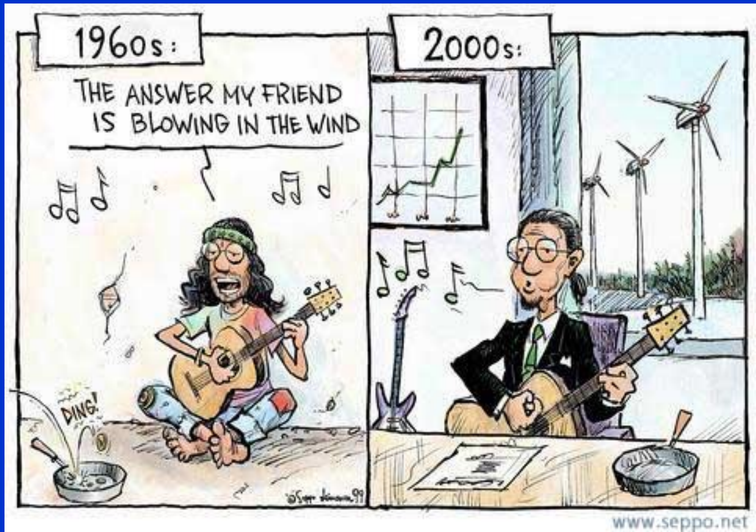
He's as blind as he can be just sees what he wants to see

- Smart meter and real-time price are key
- Solution: smart appliances
  - real time pricing, interval meters and
  - Demand-side bidding
- Large two-sided market!!!!!!!!!

New markets new technologies

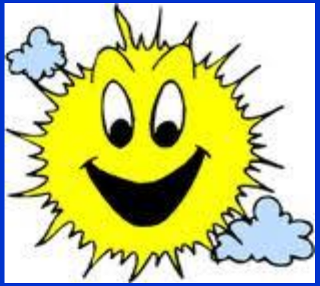
- Batteries, flexible generators, topology optimization and responsive demand
- optimally integrated
- off-peak
  - Generally wind is strongest
  - Prices as low as -$30/MWh
- Ideal for battery charging

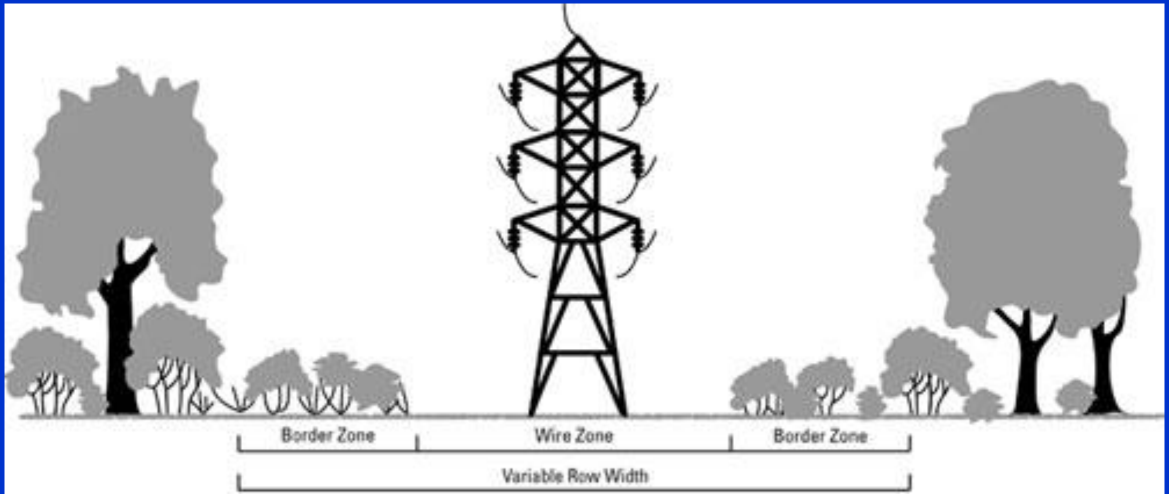




More flexible transmission markets

- Thermal capacity is similar to a storage device: manage dynamically
- relaxation penalties v dynamic management of transmission assets
- incentives



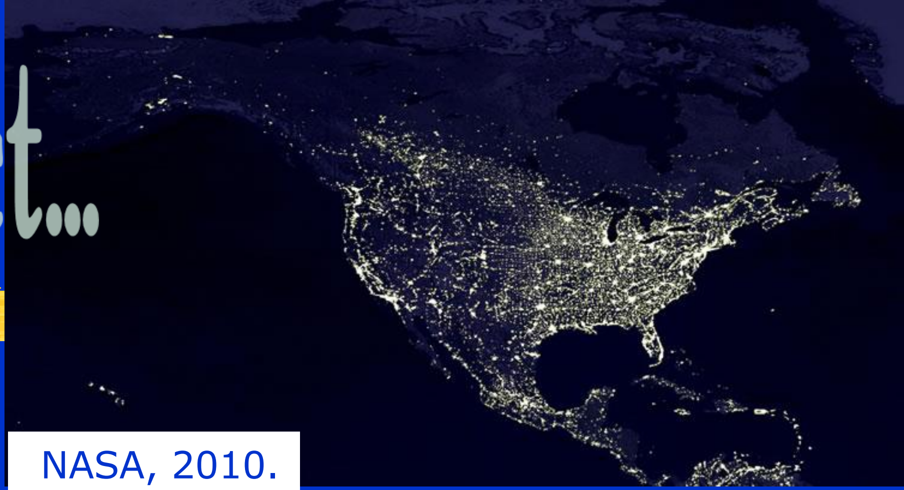


ISO Markets and Planning

- **Four main ISO Auctions**
  - Real-time: for efficient dispatch
  - Day-ahead: for efficient unit scheduling
  - Generation Capacity: to ensure generation adequacy and cover efficient recovery
  - Transmission rights (FTRs): to hedge transmission congestion costs

- **Planning and investment**
  - Competition and cooperation
- All use approximations due to software limitations

The Potential Impact…


NASA, 2010.

- World Gross Production (2009): 20,000 TWh
- United States Gross Production (2009): 4,000 TWh
- At $30/MWh: cost $600 billion/year (world)
  - cost $120 billion/year (US)
- At $100/MWh: cost $2,000 billion/year (world)
  - cost $400 billion/year (US)
- In US 10% savings is about than $10 to $40 billion/yr
- FERC strategic goal: Promote efficiency through better market design and optimization software

Source: IEA Electricity Information, 2010.

☹ **money can't buy me love**

From real time dispatch to investment planning

Mixed Integer Nonconvex Program

$$
\begin{aligned}
&\text{maximize} && c(x) \\
&\text{subject to} && g(x) \le 0, \\
& && Ax \le b \\
& && l \le x \le u, \\
& && \text{some } x \in \{0,1\}
\end{aligned}
$$

c(x), g(x) may be non-convex

I didn't know what I would find there

Mixed Integer Program

maximize $\quad cx$

subject to $\quad Ax = b,$

$l \leq x \leq u,$

some $x \in \{0,1\}$

And though the holes were rather small
They had to count them all

Better modeling for

Start-up and shutdown

Transmission switching

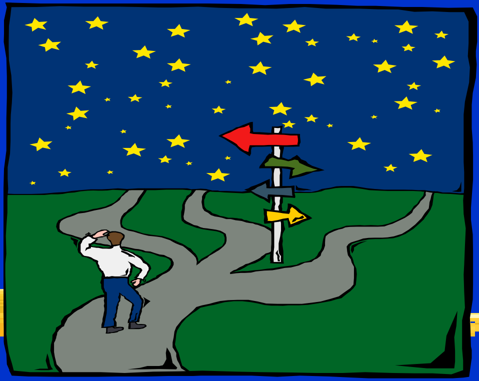
Investment decisions

It was twenty years ago today

solution times improved by > 10^7 in last 30 years

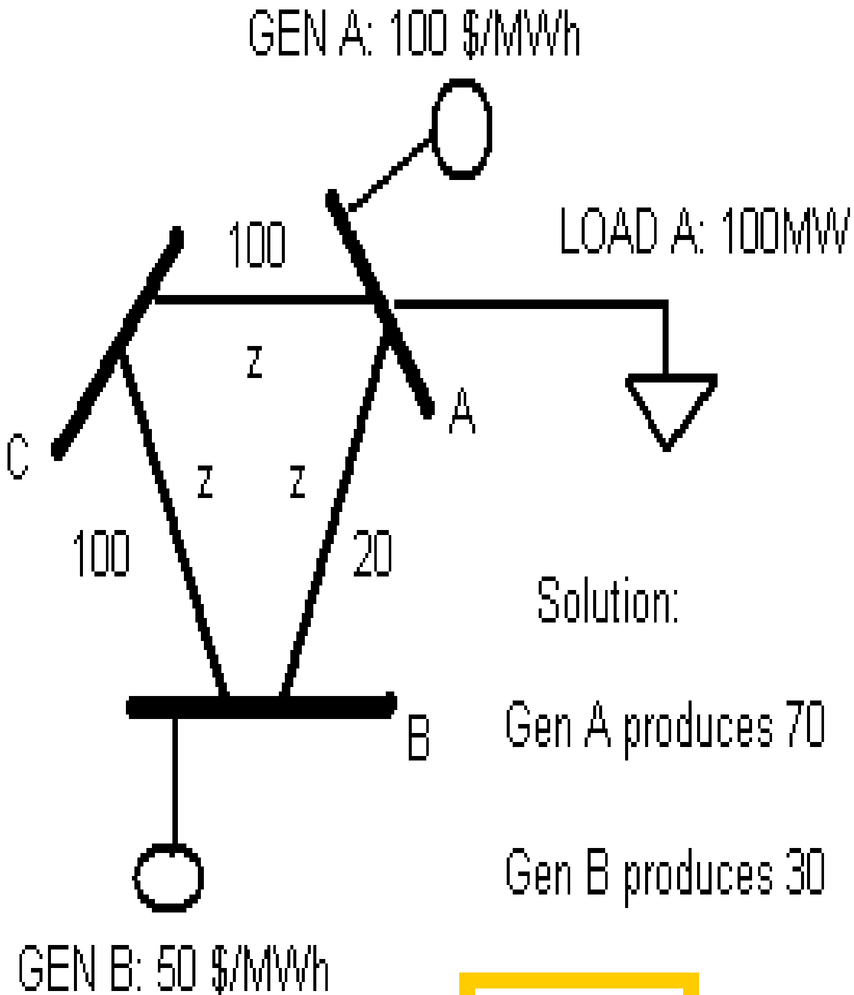
10 years becomes 10 minutes

MIP Paradigm shift:
Let me tell you how it will be

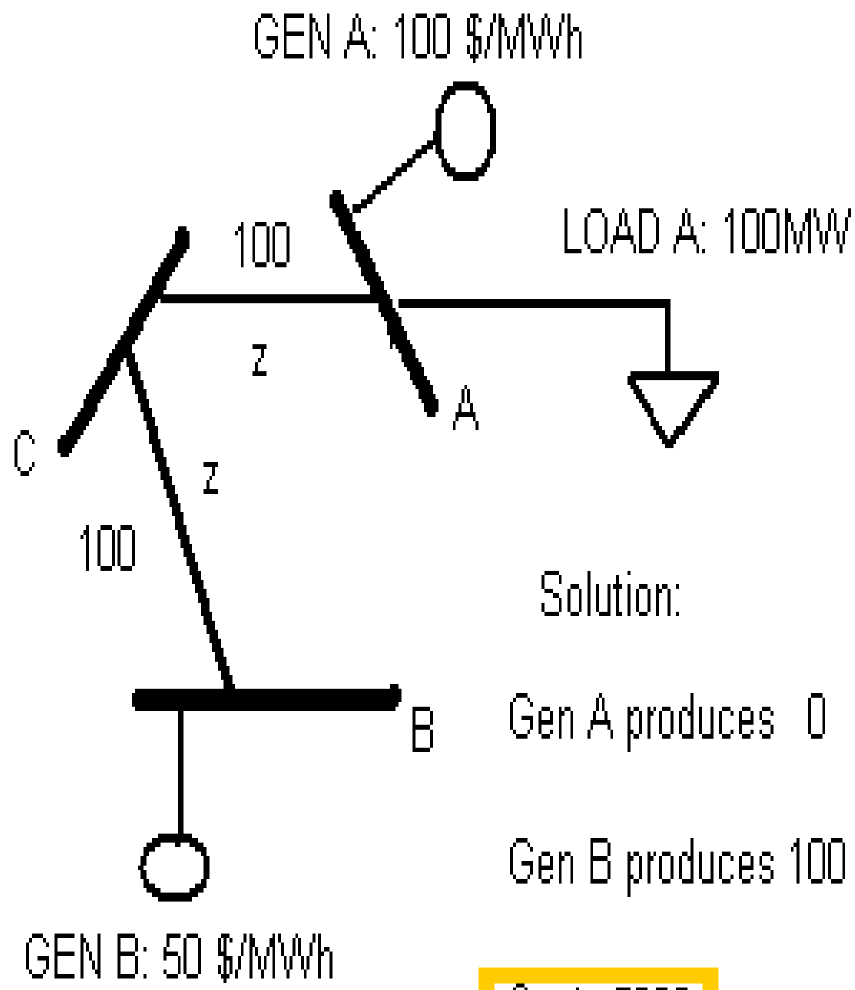


- Pre-1999
  - MIP can not solve in time window
  - Lagrangian Relaxation
    - solutions are usually infeasible
    - Simplifies generators; no switching
- 1999 Unit commitment conference and book
  - Bixby demonstrates MIP improvements
- 2011 MIP creates savings > $500 million annually
- MIP provides new modeling capabilities
  - New capabilities may present new challenges
- 2015 MIP savings of > $2 billion annually

Optimal transmission switching



Cost: 8500



Cost: 5000

Optimal Transmission Switching DCOPF Formulation

- Fisher et al IEEE 118 bus model 25% savings found.
- Hedman et al
  - ISONE 5000 bus model **13% savings**
- N-1 reliability constraints
- Hedman et al
  - IEEE 118 Bus Model 16% savings
  - IEEE 73 (RTS 96) Bus Model 8% savings

transmission switching

- Philpott: switching using column generation lowers unit commitment
- Ruiz et al: captured up-to 96% of potential cost savings with limited computational effort
- Ostrowski et al Anti-Islanding on RTS-96

TS problem w/o connectivity		with connectivity constraints		with N-1 connectivity constraints	
Time (s)	Nodes	Time (s)	Nodes	Time (s)	Nodes
524	11,306	204	2,988	32	179

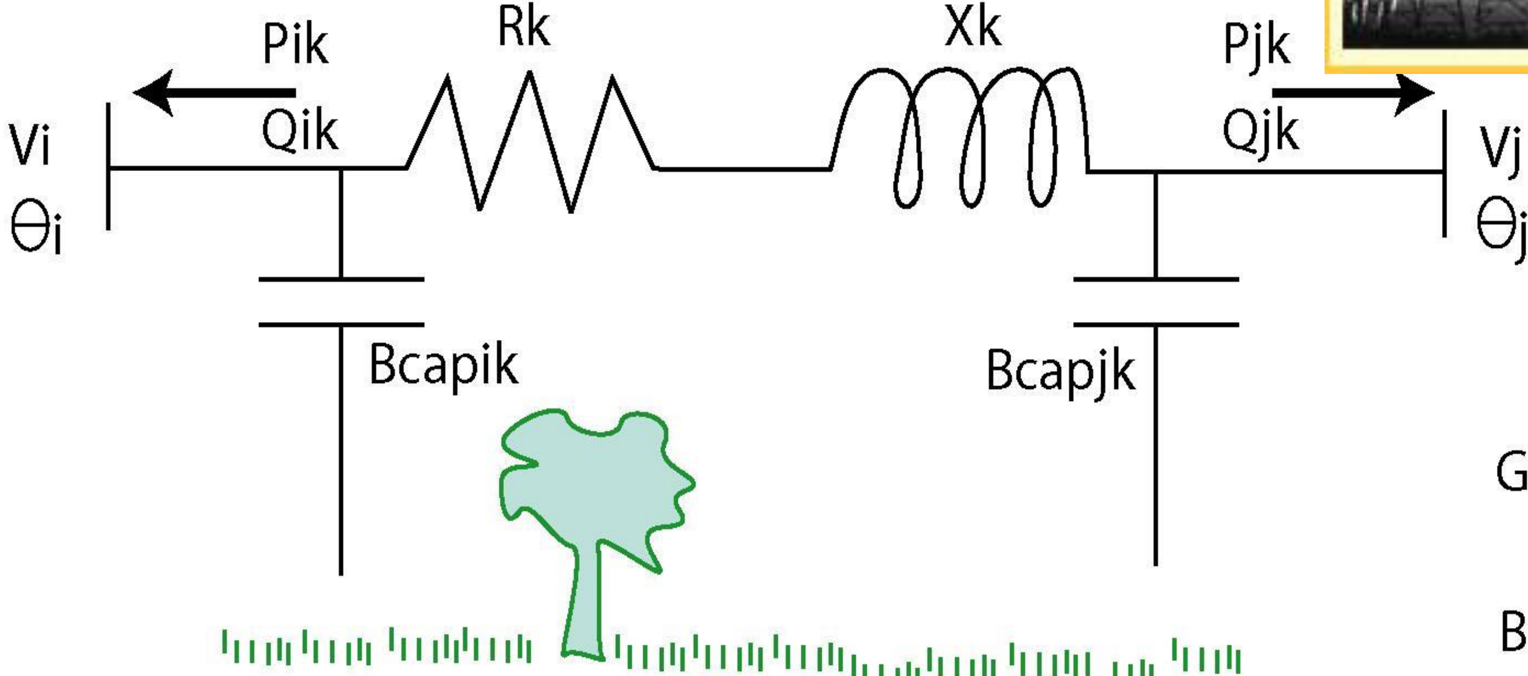
transmission switching

- 'better' solutions found 'quickly'
- In 5 years solutions are 100 times faster
- Now considered part of the smart grid
- Potential
  - solutions have optimality gaps
  - higher savings may be found
  - still takes too long to solve to optimality
  - Better solutions are acceptable
- Useful in many applications
- Next step: AC switching

transmission switching

problem	current	next decade
Corrective switching	none	Real-time
Real-time market	Pre-studied	Real-time
day-ahead market	Pre-studied	Day ahead
Maintenance scheduling	none	monthly
Optimal planning	none	annual

Power Flow and Admittance



$$G = \frac{R}{R^2 + X^2}$$

$$B = \frac{-X}{R^2 + X^2}$$

AC Model (physics)

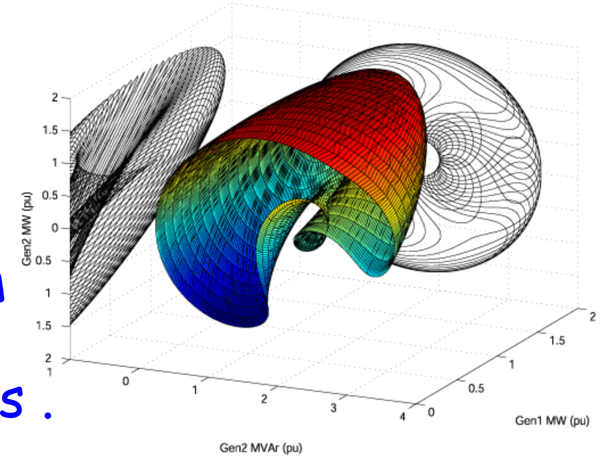
$$Pik = Gk^*Vi^2 - Gk^*(Vi^*Vj)^*\cos(\Theta i - \Theta j) - Bk^*(Vi^*Vj)^*\sin(\Theta i - \Theta j)$$

$$Qik = -Bk^*Vi^2 - Gk^*(Vi^*Vj)^*\sin(\Theta i - \Theta j) + Bk^*(Vi^*Vj)^*\cos(\Theta i - \Theta j) - Bcapik^*Vi^2$$

DC Model (market model approximation. Can we do better?)

$$Pik = -Bk^*(\Theta i - \Theta j)$$

AC Optimal Flow Problem



"DC OPF" formulations linearize the nonlinearities .

'ACOPF' formulation is a continuous nonconvex optimization problem

Most nonlinear solvers find at best local optimal solutions

Linear IV approximation to ACOPF

If promising, it can be embedded in binary formulations:

unit commitment models, and optimal topology models.

allows the use of exceptionally fast and robust MIP algorithms

Power Flow Equations

Polar Power-Voltage: 2N nonlinear equality constraints

$$P_n = \Sigma_{mk} V_n V_m (G_{nmk} \cos\theta_{nm} + B_{nmk} \sin\theta_{nm})$$

$$Q_n = \Sigma_{mk} V_n V_m (G_{nmk} \sin\theta_{nm} - B_{nmk} \cos\theta_{nm})$$

Rectangular Power-Voltage: 2N quadratic equality constraints

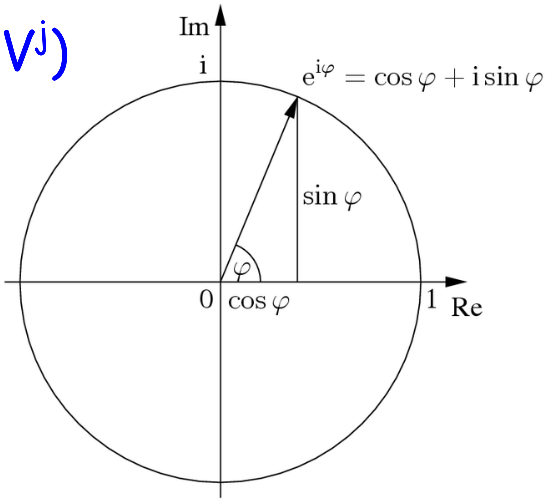
$$S = P + jQ = \mathrm{diag}(V)I^* = \mathrm{diag}(V)[YV]^* = \mathrm{diag}(V)Y^*V^*$$

Rectangular Current-Voltage (IV) formulation.

Network-wide ***LINEAR*** constraints: 2N linear equality constraints

$$I = YV = (G + jB)(V^r + jV^j) = GV^r - BV^j + j(BV^r + GV^j)$$

where $I^r = GV^r - BV^j$ and $I^j = BV^r + GV^j$



Rectangular ACOPF-IV formulation.

Network-wide objective function: Min c(P, Q, I V)) (50)

Network-wide constraint: $I = YV$ (51)

Bus-specific constraints:

$P = V^r \cdot I^r + V^j \cdot I^j \leq P^{max}$ (54) $P^{min} \leq P = V^r \cdot I^r + V^j \cdot I^j$ (55)

$Q = V^j \cdot I^r - V^r \cdot I^j \leq Q^{max}$ (56) $Q^{min} \leq Q = V^j \cdot I^r - V^r \cdot I^j$ (57)

$V^r \cdot V^r + V^j \cdot V^j \leq (V^{max})^2$ (58) $(V^{min})^2 \leq V^r \cdot V^r + V^j \cdot V^j$ (59)

$(i_{nmk})^2 \leq (i^{max}{}_k)^2$ for all k (60)

$[\theta^{min}{}_{nm} \leq \arctan(v^j{}_n / v^r{}_n) - \arctan(v^j{}_m / v^r{}_m) \leq \theta^{max}{}_{nm}$ (61)]

$V^r \geq 0$ (62)

(51) are 2N linear equality constraints that apply throughout the network,

(54) – (57) are quadratic and non-convex.

(58) are convex quadratic inequality constraints, but

(59) are non-convex quadratic inequality constraints.

(61) could be eliminated and the problem becomes quadratic with linear network equations.

Generator and Load Constraints.

The lower and upper bound constraints for generation and load are:

$P^{min} \le P \le P^{max}$ (24) $Q^{min} \le Q \le Q^{max}$ (26)

In terms of V and I,

$V^r \cdot I^r + V^j \cdot I^j \le P^{max}$ (28) $P^{min} \le V^r \cdot I^r + V^j \cdot I^j$ (29)

$V^j \cdot I^r - V^r \cdot I^j \le Q^{max}$ (30) $Q^{min} \le V^j \cdot I^r - V^r \cdot I^j$ (31)

(28)-(31) are non-convex constraints.

Voltage constraints.

in rectangular coordinates

$$(v^r{}_m)^2 + (v^j{}_m)^2 \le (v^{max}{}_m)^2$$

$$(v^{min}{}_m)^2 \le (v^r{}_m)^2 + (v^j{}_m)^2$$

voltage magnitude bounds are generally in the range, [.95, 1.05].

high voltages are often constrained by circuit breakers capabilities.

Low voltage constraints can be due operating requirements of motors or generators.

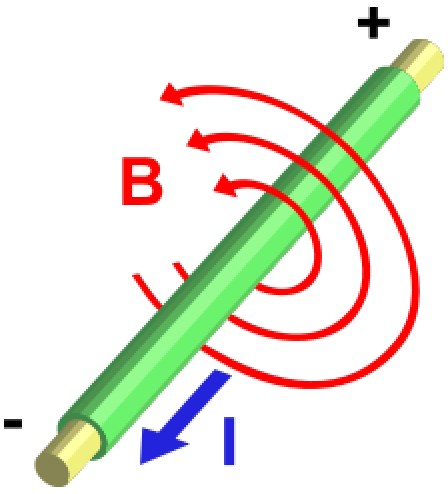
Line Flow Constraints

Power Line Flow Constraints.

$(s^r_{nmk})^2 + (s^j_{nmk})^2 = |s_{nmk}|^2 \leq (s^{max}_k)^2$ (37)

Current Line Flow Limitations.

$(i^r_{nmk})^2 + (i^j_{nmk})^2 \leq (i^{max}_{nmk})^2$ (38)



convex quadratic and isolated to the complex current at the bus.

Voltage Angle Constraints.

$\theta^{min}_{nm} \leq \theta_n - \theta_m \leq \theta^{max}_{nm}.$ (39)

(38) appears to be the best choice

The Linear Approximations to the IV Formulation

We take three approaches to constraint formulation.

If the constraint is nonlinear,

use the first order Taylor series approximation

updated at each LP iteration

If the constraint is convex,

add linear cutting planes to remove from the linear feasible

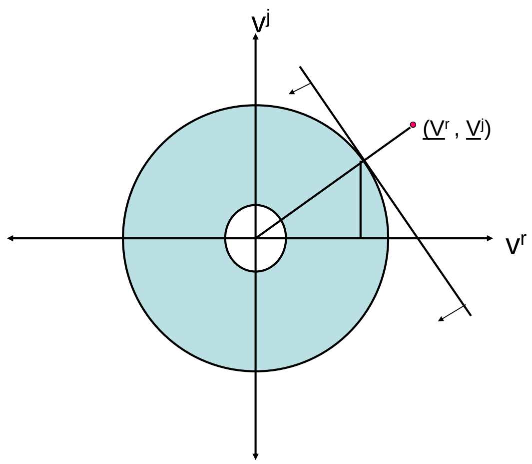
Can we guarantee feasibility with this approach?

Linear Voltage Approximations.

a first order Taylor's series approximation about $(\underline{V}^r, \underline{V}^j)$

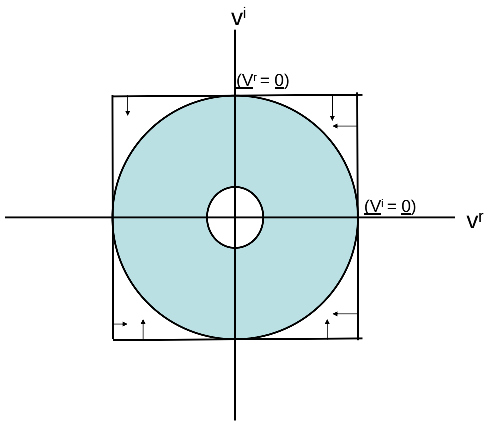
$$V^r \cdot V^r + V^j \cdot V^j \approx 2\underline{V}^r \cdot V^r + 2\underline{V}^j \cdot V^j - \underline{V}^r \cdot \underline{V}^r - \underline{V}^j \cdot \underline{V}^j$$

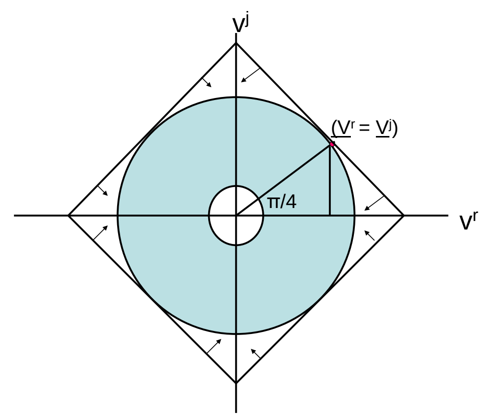
Since higher losses occur at lower voltages, the natural tendency of the optimization will be toward higher voltages.



Preprocessed Linear Voltage and Current Constraints.

$$(v^r_m)^2 + (v^j_m)^2 \le (v^{max}_m)^2$$





Current constraint set has no hole

Iterative Voltage and Current Constraints.

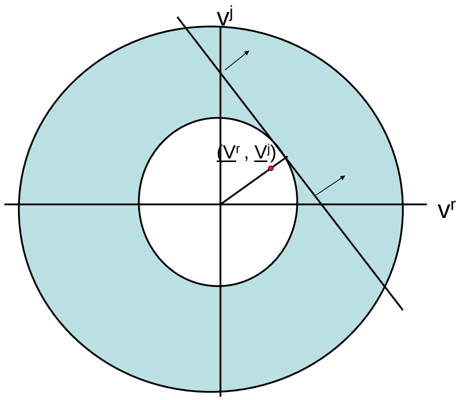
Adding a maximum-voltage linear constraint.

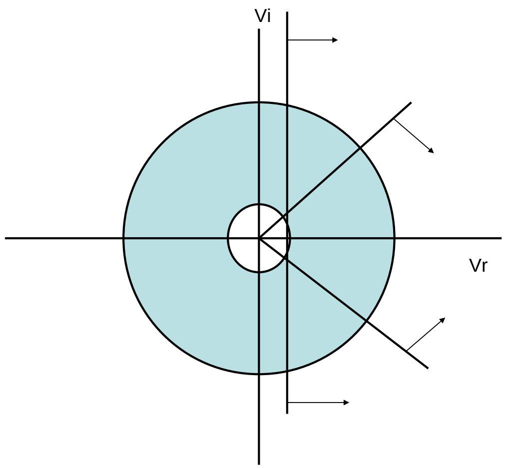
Non-Convex Minimum Voltage Constraints.

$(v^{min}{}_m)^2 \le (v^r{}_m)^2 + (v^j{}_m)^2$

non-convex, the linear approximation is problematic.

approximation and eliminates parts of the feasible region





This is probably not a good idea, but maybe.

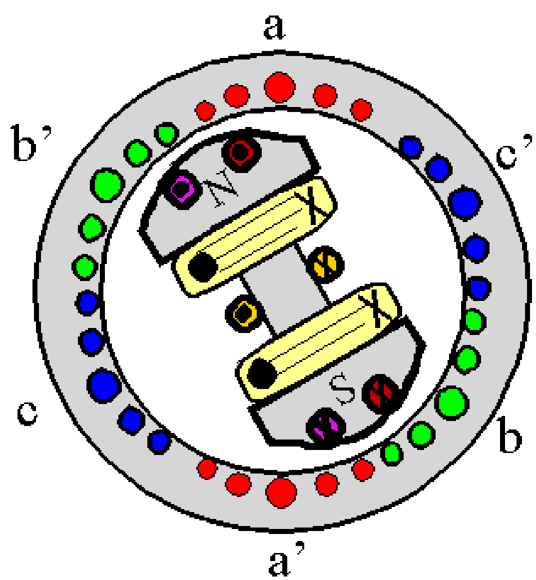
Real Power Constraints. At each bus

first order approximation at bus n around $\underline{v}^r_n$, $\underline{i}^r_n$, $\underline{v}^j_n$, $\underline{i}^j_n$

$$p^{\approx}_n = \underline{v}^r_n i^r_n + \underline{v}^j_n i^j_n + v^r_n \underline{i}^r_n + v^j_n \underline{i}^j_n - (\underline{v}^r_n \underline{i}^r_n + \underline{v}^j_n \underline{i}^j_n)$$

hessian is

$$\begin{matrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{matrix}$$



eigenvalues: 2 are 1 and 2 are -1

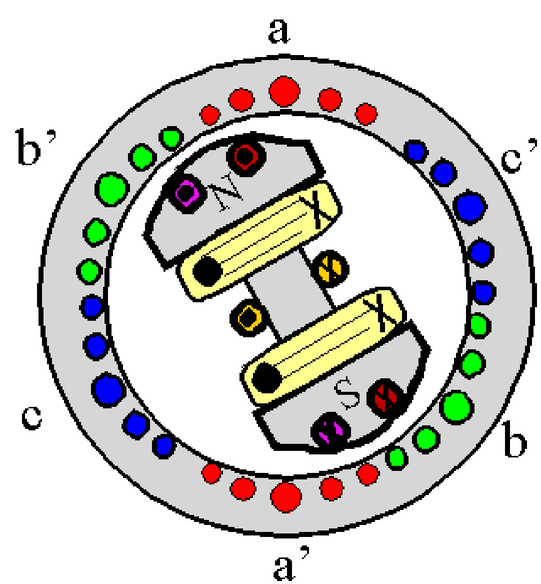
Reactive Power Constraints. At each bus

First order approximation around $\underline{v}^r_n$, $\underline{i}^r_n$, $\underline{v}^j_n$, $\underline{i}^j_n$

$$\tilde{q}_n = v^j_n i^r_n - v^r_n i^j_n - v^r_n \underline{i}^j_n + v^j_n \underline{i}^r_n + (\underline{v}^j_n \underline{i}^r_n - \underline{v}^r_n \underline{i}^j_n)$$

The Hessian is

$$\begin{matrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{matrix}$$



Eigenvalues: 2 are 1 and 2 are -1.

Computational experience

- Naïve approximation
  - IV SLP faster or competitive with commercial non-linear solvers
- Add transmission constraints and tune algorithm parameters,
  - greater speedups and accuracy
- rectangular formulations solve faster then polar
  - maybe function evaluations
- Switching approximation much faster

Computational Research Questions

- Decomposition and Grid (parallel) computing
  - Real/reactive
  - Time
- Good approximations
  - Linearizations
  - convex
- Avoiding local optima
- Nonlinear prices
- Better tree trimming
- Better cuts
- Advance starting points

If you really like it you can have the rights
It could make a million for you overnight